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Convolution in laplace domain

In this section we consider the problem of finding the inverse Laplace transformation of a product \ (H (s) =F (s) G (s) \), where \ (F\) and \ (G\) are the Laplace transformations of the known functions \ (f\) and \ (g\). To justify our interest in this problem, consider the problem of the initial value \[ay " +by'+cy=f (t),\quad y (0) =0,\quad y' (0) =0.onumber \] The Laplace assumption transforms the yields \[(as^2+bs+c) Y (s) =F (s),onumber \] so \[\label{eq:8.6.1} Y (s) =F (s) G (s),\] where \[G (s) ={1\over as^2+bs+c}.onumber \] Until now we have not been interested in the factoring given in Equation \ref{eq:8.6.1}, as we have only treated differential equations with specific functions. forcing. So, we could just do the multiplication given in Equation \ref{eq:8.6.1} and use the Laplace transform table to find \ (y={\cal L}^{-1} (Y) \). However, this is not possible if we want a formula for \ (y\) in terms of \ (f\), which can be unspecified. To justify the formula for \ ({\cal L}^{-1} (FG) \), consider the problem of the initial value \ [\label{eq:8.6.2} y'-ay=f (t),\quad y (0) =0,\] which we solve first without using the Laplace transformation. The solution of the differential equation in Equation \ref{eq:8.6.2} is of the form \ (y=ue^{at}\) where \[u'=e^{-at}f (t).onumber \] The integration from \ (0\) to \ (t\) and the imposition of the initial condition \ (u (0) =y (0) =0\) produces \[u=\int_0^t e^{-a\tau}f (\tau) \,d\tau.onumber \] Therefore, \[\label{eq:8.6.3} y (t) =e^{at}\int_0^t e^{-a\tau}f (\tau) \,d\tau=\\\int_0^ t e^{a (t-\tau) }f (\tau) \,d\tau\] We will now use the Laplace transformation to solve the equation \ref{eq:8.6.2} and compare the result to Equation \ref{eq:8.6.3}. Taking Laplace turns to Equation \ref{eq:8.6.2} produces \[(s-a) Y (s) =F (s),onumber \] so \[Y (s) =F (s) {1\over s-a},onumber \] which implies that \[\label{eq:8.6.4} y (t) = {\cal L}^{-1}\left (F (s) {1\over s-a}\right).\] If we now leave \ (g (t) =e^{at}\), so that \[G (s) ={1\over s-a},onumber \] then Equation \ref{eq:8.6.3} and Equation \ref{eq:8.6.4} can be written as \[y (t) =\int_0^tf (\tau) g (t-\tau Therefore \[\label{eq:8.6.5} {\cal L}^{-1} (FG) =\int_0^t f (\tau) g (t-\tau) \,d\tau\] in this case. This is the reason for the next definition. Definition 8.6.1: Convolution The convolution \ (f*g\) of two functions \ (f\) and \ (g\) is defined by \[(f*g) (t) =\int_0^t f (\tau) g (t-\tau) \,d\tau.onumber \] It can be shown (Exercise 8.6.6) that \ (f\ast g=g\ast f\); i.e. \[\int_0^tf (t-\tau) g (\tau) \,d\tau=\tf (\tau) g (t-\tau) \,d\tau\. This later theorem states that this is generally true. Theorem 8.6.2: The convolution theorem if \ ({\cal L} (f) =F\) and \ ({\cal L} (g) =G,\) then \[{\cal L} (f*g) =FG.onumber \ A complete proof of the convolution theorem is beyond the scope of this book. However, suppose that \ (f\ast g\) has a Laplace transformation and verifies the conclusion of the theorem into a pureVery. From the definition of the Laplace transform, \[{\cal L} (f\ast g) =\int_0^\infty e^{-st} (f\ast g) (t) \,dt=\int_0^\infty e^{-st} \int_0^t f (\tau) g (t-\tau) \,d\tau\,dt.o (a) the Solution First we calculate \[\begin{aligned} (f\ast g) &= \int_{0}^{t}e^{a\tau }e^{b (t-\tau) }d\tau &=e^{bt}\int_{0}^{t}e^{ (a-b) \tau }d\tau \ &= \sinistra (b) a The next three examples illustrate it. Example 8.6.2 Find a formula for the solution of the problem of the initial value \[\label{eq:8.6.7} y « -2y'+y=f (t),\quad y (0) =k_0,\quad y' (0) =k_1.\] Solution Taking Laplace turns into the equation \ref{eq:8.6.7} returns \[(s^2-2s+1) Y Then \[\begin{align*} (s) (s) (s-1) (s) (s) (s)) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s)) (s) (s) (s)) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s) (s)))) (s) (s) (s) (s)) (s) (s) (s) (s)))) (s) (s number\ From \[{1\over (s-1) ^2}tarrow of left te^t\quad \text{and} \quad F (s) \tarrow of left f (t), number\] the convolution theorem implies that \[{\cal L}^{-1} \left ({1\over (s-1) ^2}F (s) \right) =\tauthe solution of the equation \ref{eq:8.6.7} is \[y (t) =e^t\left (k_0+ (k_1-k_0) t\right) +\int_0^t\tau e^\tau f (t-\tau) \, d\tau onumber\] Example 8.6.3 Find a formula for solving the problem of the initial value \ [\label{eq:8.6.8} \[\label{eq:8.6.8}y (0) =k_0,\quad y' (0) =k_1.\] solution taking laplace is transformed into the equation \ref{eq:8.6.8} returns \[(s^2+4) y (s) =f (s) +k_1+k_0s. number\] then [\y (s) ={1\over (s^2+4) }f (s) +{k_1+k_0s\over s^2+4}. (b) the number of the == sync, corrected by elderman == (b) the number of the We say that an integral of the \ form (\int_0^t u (\tau) v (t-\tau) \,d\tau\) is an integral of convolution. the convolution theorem provides a convenient way to assess the whole of convolution. example 8.6.5 evaluates the integral of convolution [\h (t) =\int_0^t (t-\tau) ^5\tau^7 d\tau. onumber\] solution we could evaluate this integral by expanding \ (t-\tau) ^5\) into \ powers and then integrating it. However, convolution theorem provides a simpler way. The integral is the convolution of \ (f (t) =t^5\) and \ (g (t) =t^7\). by \[t^5\leftrightarrow {5!\over s^6}\quad\mbox{ and \}quad t^7 \leftrightarrow {7!\over s^8},number \] theorem of convolution implies that [\h (t) \leftrightarrow {5!7!\over s^14} {5!7!\over 13\ {13! \over s^{14}},number \] where we wrote the second equity because \[{13!\over s^{14}}\leftrightarrow t^{13}.number \] therefore, [\h (t) ={5!7!\over 13\!,} t^{13}. number \] example 8.6.6 use convolution theorem and a partial expansion of the fraction to evaluateof CONVOLUTION \ [H (T) = \ int_0 ^ t \ t- \ tau) \ cos b \ tau \, d \ tau \, s) (| e | b |). Number \] Solution from \ [Sin At \ LEFTROW ^ 2 + A ^ 2} \ s quail \ s) \ s) \ s quot \ bt \ left ♪ The convolution theorem implies that \[H(s)={a\over s^2+a^2}{s\over s^2+b^2}.onumber \] The expansion of this product in a partial fraction produces \[H(s)={a\over b^2-a^2}\left[{s\over For this reason, the number of people who are 2 years old is 2 years old. An equation of the form \[\label{eq:8.6.11} y(t)=f(t)+\int_0^t k(t-\tau) y(\tau)\,d\tau\] is an integral equation of Volterra. Here \(f\) and \(k\) are given functions and \(y\) is unknown. Since the integral on the right is a convolution integral, the convolution theorem provides an affordable formula to solve the equation \ref{eq:8.6.11}. Take Laplace transforms into Equation \ref{eq:8.6.11} produces \[Y(s)=F(s)+K(s) Y(s),onumber \] and solve it for \(Y(s)\) produces \[Y(s)={F(s)\over 1-K(s)}.onumber \] Then we obtain the solution of Equation \ref{eq\ ) In Exercises 8.6.11-8.6.14 it is shown that the formula is valid under much weaker conditions on \(f\). Theorem 8.6.3 Suppose \(f\) is continuous on \([0,\infty)\) and has a Laplace transformation. (i) Proof Taking Laplace turns into Equation \ref{eq:8.6.13} produces \[p(s)Y(s)=F(s)+a(k_1+k_0s)+bk_0,onumber \] where \[p(s)=as^2+bs+c.onumber \] For this reason, with the help of a group of experts, the Taking Laplace turns into Equation \ref{eq:8.6.15} and Equation \ref{eq:8.6.16} shows that \[p(s)Y_1(s)=as+b\quad\mbox{e}\quad p(s)Y_2(s)=a.onumber \] For this reason, the number of people who have no children is no longer Therefore, the \ref{eq:8.6.20} equation can be rewritten as \[V(s)=k_0Y_1(s)+k_1Y_2(s).onumber \] Replace this in Equation \ref{eq:8.6.18} produces \[Y(s)=k_0Y_1(s)+k_1Y Finally, the equationRef {EQ: 8.6.21} They imply the equation ref {EQ: 8.6.17}. It is useful to note from equation ref {eq: 8.6.14} which (y) is the form [y = v + h, onumber where [v (t) = k_0y_1 (t) + k_1y_2 (t) Onumber Depends on the initial conditions and is independent of the forcing function, while (t) = if the zeros of the polynomial feature [P (s) = AS ^ 2 + BS + CONMER Complementary have negative real parts, then (Y_1) and (Y_2) Both approach zero as (t), then ({t = 0) for any choice of initial conditions. Furthermore, the value of (H (T) is essentially independent of the values of (T- TAU) for large (TAU), from the (LIM _ {TAU INFTY} W (TAU) = 0). In this case we say that (V) are components of transient and constant state, respectively, of the solution (y) of equation ref {eq: 8.6.13}. These definitions apply to the problem of the initial value of the example 8.6.4, where the zeros of [P (s) = s ^ 2 + 2 = (s + 1) ^ 2 + 1onumber are (- 1 PM I). From equation ref {eq: 8.6.10}, we see that the solution of the general problem of the initial value of the example 8.6.4 is (y = v + h), where [v (t) = e ^ {-T} Left (K_1 + K_0) Intsin T + K_0 COS T Right) Onumber is the transient component of the solution and T (the definitions do not apply to initial value problems considered in Examples 8.6 .2 and 8.6.3, since the zeros of the characteristic polynomes in these two examples have no negative real parts. In the physical applications where the input (f) and the output (y) of a device are Connected by equation ref {EQ: 8.6.13}, the zeros of the characteristic polynomial usually have negative real parts. Then (w = {cal} (W)) is called the transfer function of the device. Since [h (s) = w (s) f (s), onumber we see that [s) = {h (s) over f (s)} Onumber is the report Among the processing of the output of the constant state to the transformation of the entrance. Due to the form of [H (T) = INT_0 ^ TW (TAU) F (T- TAU), D TAU, ONUMBER (W) is sometimes called the weighting function of the Device, since it assigns weights to the passed values of the input (F). The device's impulse response is also called, for reasons discussed in the next section. Formula Equation Ref {EQ: 8.6.14} is more detailed in the years 8.6.8-8.6.10 for the three possible cases in which the zeros of (s) are real and distinct, real and Repeated, or complex conjugates, respectively. respectively.

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